Cuckoo Search Algorithm with Lévy Flights for Reconstruction of Periodic and Chaotic Orbits of the 2D Hénon Map

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ABSTRACT

Reconstructing the complex behavior of chaotic dynamical systems from time series is a challenging task, due to the high sensitivity to initial conditions and other factors. In this paper, we formulate this issue as an optimization problem, which is addressed through a bio-inspired swarm intelligence technique: the cuckoo search algorithm with Lévy flights. The method is applied to reconstruct several periodic and chaotic behaviors of the Hénon map, a popular two-dimensional chaotic map. Our graphical and numerical results show that the proposed method is able to obtain suitable values for the parameters under optimization and reconstruct different system behaviors with good accuracy.

CCS CONCEPTS

• **Computing methodologies** → **Artificial intelligence**; **Continuous space search**; *Continuous simulation.*

KEYWORDS

Dynamical systems, swarm intelligence, cuckoo search algorithm, attractor reconstruction, chaotic behavior

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1 INTRODUCTION

Dynamical systems provide a powerful mathematical framework for studying the time-dependent behavior of many systems that can be described through a set of rules or equations [2, 25]. This make them a valuable tool in understanding, predicting and controlling real-world phenomena across several scientific and engineering fields. Dynamical systems can display a wide range of behaviors (steady-states, periodic oscillations, chaotic patterns, bifurcations, crises, and more) and often exhibit attractors, representing stable states where the system tends to evolve over time [15]. This evolution can be analyzed through the trajectories of the system variables, called orbits [1, 3]. Analyzing such orbits is essential for understanding dynamical systems, but it poses a significant challenge, particularly for chaotic systems [12, 26]. Various extensions and alternative techniques have been explored in the literature to address this issue [9, 20], including autoencoders for learning embeddings [16, 22], and variational Bayes filters [17]. Recently, artificial intelligence approaches have been applied to tackle this issue, including kernel methods [23], support vector machines [21], neural networks [13], machine learning [5, 18], and deep learning [19].

This paper addresses the problem of reconstructing periodic and chaotic orbits of dynamical systems using time series data. Of course, this problem is too general to be solved in just a single paper. In this work, we focus on the case of low-dimensional discrete dynamical systems. The problem is formulated as a continuous nonlinear optimization problem, which is solved using the cuckoo search algorithm with Lévy flights, a popular nature-inspired swarm intelligence method for optimization. To analyze the performance of the proposed method, it is applied to the reconstruction of several periodic and chaotic behaviors of a two-dimensional map called the Hénon map. The experimental results show that the method performs well in reconstructing different system behaviors with good accuracy.

The structure of this paper is as follows: in Sect. 2 we describe the optimization problem along with the cuckoo search algorithm, the swarm intelligence technique used in this work. Sect. 3 describes the computational experiments, introducing the example used in this paper and the implementation details. The experimental results are discussed in Sect. 4. The conclusions and some ideas for future work in the field in Sect. 5 close the paper.

2 THE PROPOSED METHOD

2.1 Problem Statement

The problem to be solved can be stated as follows: as the initial input, we are given a collection of two-dimensional data points, $\{x_n, y_n\}_{n=1,\ldots,N}$ assumed to come from a periodic or chaotic orbit of an unknown chaotic dynamical system M. The objective is to recover the periodic or chaotic orbit of the chaotic map M. In this paper, we assume that the map M can be expressed as a linear combination of bivariate vector functions from a given family $\{\phi_1(u,v), \phi_2(u,v), \ldots, \phi_m(u,v)\}$, i.e.:

$$M(u,v) = \sum_{k=1}^{m} \lambda_k \phi_k(u,v) \quad (1)$$

where the λ_k are parametric vectors to be computed. This means that $M(u_i, v_i) = (x_i, y_i)$, for all $i = 1, \ldots, N$. This problem can be expressed as such of computing the parameters of the linear combination that minimize the error between the original data and the predicted data according to the selected mathematical model. This leads to the functional:

$$\begin{aligned} \Xi &= min \left[\sum_{i=1}^{N} ||M(u_i,v_i) - (x_i,y_i)||_2 \right] \\ &= min \left[\sum_{i=1}^{N} \left\| \sum_{k=1}^{m} \lambda_k \phi_k(u_i,v_i) - (x_i,y_i) \right\|_2 \right] \end{aligned} \quad (2)$$

which is a nonlinear continuous optimization problem as long as the functions $\phi_k(u,v)$ are nonlinear. In this work, we consider the family of quadratic polynomial functions. Therefore, the model M of Eq. (1) can be expressed as:

$$M(u,v) = \lambda_1 + \lambda_2 u + \lambda_3 v + \lambda_4 u^2 + \lambda_5 v^2 + \lambda_6 uv \quad (3)$$

To solve the minimization problem in Eqs. (2)-(3) we apply a popular swarm intelligence approach called cuckoo search algorithm, which is briefly described in the next section.

2.2 The cuckoo search algorithm

The minimization problem described in Sect. 2.1 is very difficult to solve using classical mathematical techniques. Hence, the researchers have turned their attention to artificial intelligence techniques, which have been successfully applied to many challenging problems. In this paper, we consider a popular nature-inspired swarm intelligence technique named *cuckoo search algorithm* (CSA), introduced in 2009 by Xin-She Yang and Suash Deb to solve continuous optimization problems [27, 28].

This algorithm is based on the reproductive behavior of cuckoo birds, leading to some idealized rules as the basis for a computational optimization model. The corresponding pseudocode is shown in Table 1. The method starts with an initial population of N_p host nests generated randomly, and runs iteratively. At each iteration t, a cuckoo egg $\mathbf{x}_i^t$, representing a potential solution, is selected randomly and new solutions $\mathbf{x}_i^{t+1}$ replace the current ones with a probability proportional to their fitness. This step, designed to explore the search space, is performed more efficiently by using Lévy flights, a type of random walk in which the steps are defined in terms of the step-lengths following a certain probability distribution given by:

$$\mathbf{x}_i^{t+1} = \mathbf{x}_i^t + \alpha \oplus levy(\lambda) \quad (4)$$

Table 1: Original Cuckoo Search Algorithm with Lévy flights proposed by Yang and Deb in [27, 28].

Algorithm: Cuckoo Search via Lévy Flights

```
begin
  Objective function f(x), x = (x_1, ..., x_d)^T
  Generate initial population of N_p host nests x_i
  while (t < Max_gen) or (stop criterion)
    Get a cuckoo (say, i) randomly
    Evaluate its fitness F_i
    Choose a nest among N_p (say, j) randomly
    if (F_i > F_j)
       Replace j by the new solution
    end
    A fraction (p_a) of worse nests are abandoned and
          new ones are built via Lévy flights
    Keep the best solutions (or nests with quality solutions)
    Rank the solutions and find the current best
  end while
  Results postprocessing and visualization
end
```

where the symbol $\oplus$ indicates the entry-wise multiplication, $\alpha > 0$ indicates the step size, and the Lévy distribution is given by:

$$levy(\lambda) \sim g^{-\lambda}, \qquad (1 < \lambda \leq 3) \quad (5)$$

(see [27] for details). Then, the fitness of the new solution is computed and compared with the value of the current one. In case of improvement, this new solution replaces the current one. Also, a fraction of the worse nests (determined by a probability value p_a, a parameter of the method) is randomly removed and replaced by new solutions to promote global exploration. Then, all solutions are ranked according to their fitness and the best solution so far is stored as the vector $\mathbf{x}_{best}$. This procedure is repeated iteratively for a maximum number of iterations, another parameter of the algorithm. The best solution at last iteration is selected as the final solution of the optimization problem.

3 COMPUTATIONAL EXPERIMENTS

3.1 Illustrative example: the Hénon map

To analyze the performance of the proposed method, it has been applied to some chaotic systems. However, only one illustrative example is reported here because of limitations of space. The example consists of the two-dimensional Hénon map, one of the most popular examples of discrete chaotic dynamical systems [11]. It was firstly introduced by M. Hénon in [10] as a simplified model of the Poincaré section of a continuous dynamical system called the Lorenz model. It is mathematically described as a 2D map:

$$\begin{cases} x_{n+1} = & 1 - a x_n^2 + y_n \\ y_{n+1} = & b x_n \end{cases} \quad (6)$$

This map has been extensively analyzed in several previous papers, such as [4, 14]. In [24], a broad range of behaviors was reported as

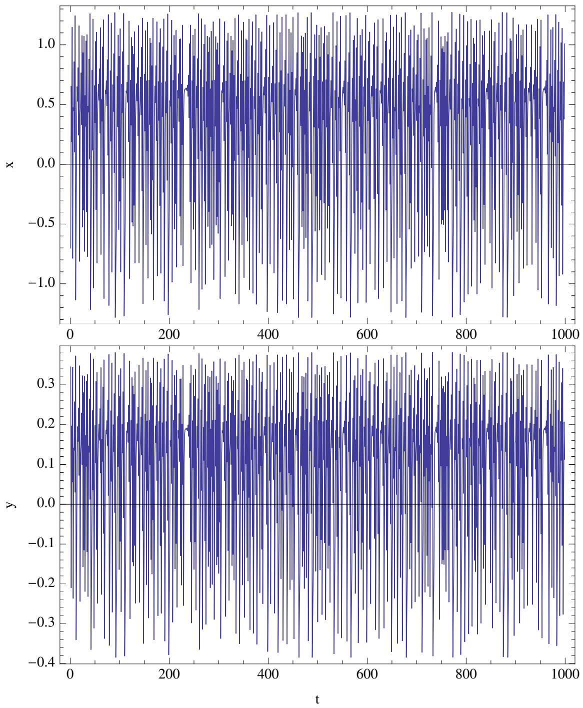

Figure 1: Time series for the variables x (top) and y (bottom) of the Hénon map for $a = 1.4$ and $b = 0.3$ showing the chaotic behavior.

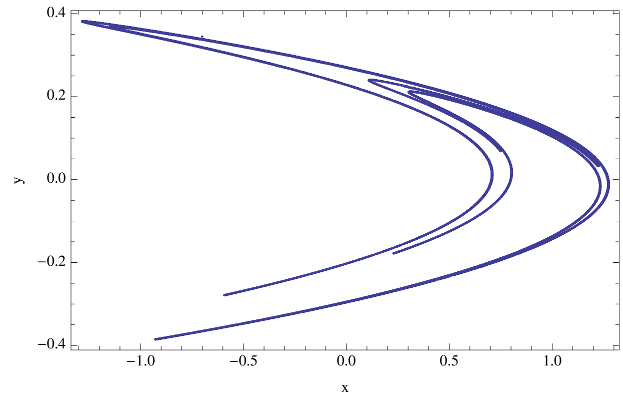

Figure 2: Chaotic attractor of the Hénon map for $a = 1.4$ and $b = 0.3$.

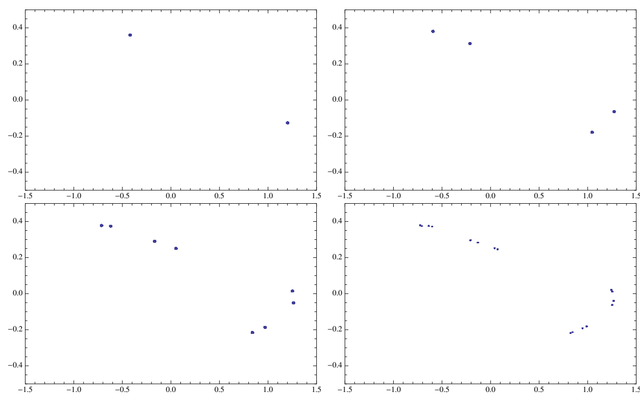

Figure 3: Period-doubling cascade of the Hénon map (Top-bottom, left-right): period-2 for $a = 0.9$; period-4 for $a = 0.95$; period-8 for $a = 1.05$; and period-16 $a = 1.055$.

a function of the parameter a, for the fixed parameter value $b = 0.3$. A classical choice for the parameter value of a is $a = 1.4$, for which the system exhibits deterministic chaos, as shown in Fig. 1, which displays 1,000 data points from the time series for the variables x and y (top and bottom pictures, respectively). Fig. 2 depicts the corresponding two-dimensional chaotic attractor for this specific value. Of course, other interesting behaviors can also be observed for this system [6–8]. In particular, a periodic behavior is obtained for the parameter value $a = 0.9$, where a period-2 orbit (shown in Fig. 3 (top-left)) is observed. Increasing the values of parameter a, the system undergoes a period-doubling cascade, with a period-4 orbit for $a = 0.95$, a period-8 orbit for $a = 1.05$ and a period-16 orbit for $a = 1.055$, shown in Fig. 3 (top-right), (bottom-left) and (bottom-right), respectively.

3.2 Method and Implementation

We applied the cuckoo search algorithm with Lévy flights, described in Sect. 2.2, to the reconstruction of the periodic and chaotic behaviors described in Sect. 3.1 from time series data. To that purpose, we need to address the following issues:

3.2.1 Population representation. To apply the cuckoo search algorithm to the optimization problem described in Sect. 2.1, we need a suitable representation for the individuals of the population. In this work, we consider a population of N_p individuals $\mathbf{x}_i$ given by a real-valued vector of $2m$ components for the parametric vectors $\lambda_{i,k} = (\lambda^x_{i,k}, \lambda^y_{i,k})$, for $k = 1, \dots, m$. Therefore, $\mathbf{x}_i$ becomes:

$$\mathbf{x}_i = (\lambda^x_{i,1}, \lambda^y_{i,1}, \dots, \lambda^x_{i,m}, \lambda^y_{i,m}) \tag{7}$$

for $i = 1, \dots, N_p$.

3.2.2 Parameter tuning. As it is well-known, the performance of the metaheuristic techniques is usually affected by the choice of suitable values for their parameters. In this context, the cuckoo search algorithm is particularly advantageous, as it only requires three parameters:

- the population size N_p,
- the probability p_a, and
- the maximum number of iterations, Max_{gen}.

In this work, a population of $N_p = 100$ host nests, representing the number of candidate solutions for the method, is considered. Regarding the parameter p_a, after several trials, its value is empirically selected as $p_a = 0.25$, which reduces the number of iterations required for convergence. Finally, we select $Max_{gen} = 10,000$ iterations, which has been found enough to reach convergence of the method in all cases.

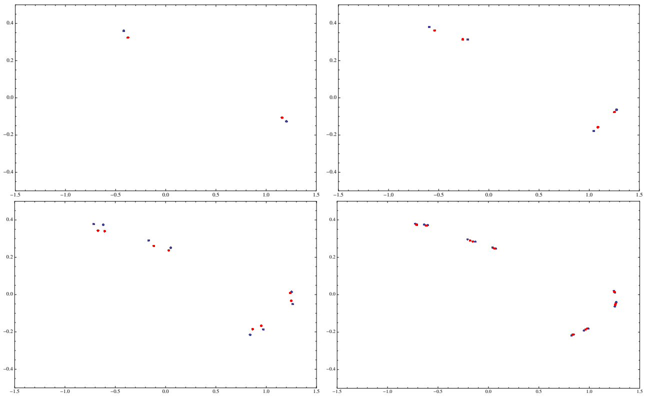

Figure 4: Original (blue) and reconstructed (red) periodic orbits of the Hénon map: (top-left) period-2;(top-right) period-4; (bottom-left) period-8;(bottom-right) period-16.

3.2.3 Fitness function. The functional in Eq. (2), based on the Euclidean distance between time series, can be used to address the reconstruction problem considered in this paper. However, this fitness function does not consider the size of the time series, and hence, the longer the time series, the larger the error. In order to take the length of time series into account, it is convenient to consider the RMSE (root-mean-square error), given by Eq. (8):

$$RMSE = \sqrt{\sum_{i=1}^{N} \frac{(\mathbf{x}_i - \bar{\mathbf{x}}_i)^2}{N}} \tag{8}$$

where $\mathbf{x}_i$ and $\bar{\mathbf{x}}_i$ denote the observed and the predicted values respectively, and N indicates the sample size.

3.2.4 Computational issues. The computations in this paper have been carried out on a PC desktop with a processor Intel Core i9 running at 3.7 GHz and with 64 GB of RAM. The source code has been implemented by the authors in the programming language of the popular scientific program *Mathematica* version 12.

4 RESULTS

The goal in this paper is to obtain suitable parametric values for the model M according to Eq. (3) so that the observed periodic and chaotic behaviors can be accurately reconstructed. This is achieved through optimization according to Eq. (2). To this purpose, the cuckoo search algorithm described in Sect. 2.2 is applied. The potential solutions follow the continuous representation described in Sect. 3.2.1 for the parameter tuning described in Sect. 3.2.2.

An important observation from our simulations is that the values of some parameters obtained by solving the optimization problem are quite small, meaning that their contribution to the model is not significant, but still adding unnecessary complexity to the model. To address this issue, we removed all terms which coefficients are smaller than a threshold value, selected empirically as 10^{-2} in this paper, so that the corresponding terms of the equation are not actually considered. In our trials we found that this strategy improved the accuracy of the model and reduced its complexity, so we integrated it as part of our method. With this modification,

Table 2: Best results for the non-null parameter values and RMSE of the reconstructed periodic orbits in Fig. 4.

Period	*Non-null parameter values*	*RMSE*
2	$\lambda_1^x = 0.97$, $\lambda_3^x = 0.988$, $\lambda_4^x = -0.93$, $\lambda_2^y = 0.286$	5.1761E−2
4	$\lambda_1^x = 0.993$, $\lambda_3^x = 0.992$, $\lambda_4^x = -0.94$, $\lambda_2^y = 0.291$	4.5378E−2
8	$\lambda_1^x = 0.994$, $\lambda_3^x = 1.0103$, $\lambda_4^x = -1.0573$, $\lambda_1^y = 0.016$, $\lambda_2^y = 0.275$	5.1805E−2
16	$\lambda_1^x = 1.0126$, $\lambda_3^x = 0.996$, $\lambda_4^x = -1.054$, $\lambda_1^y = 0.0206$, $\lambda_2^y = 0.2973$	2.6538E−2
7	$\lambda_1^x = 1.013$, $\lambda_3^x = 1.024$, $\lambda_4^x = -1.252$, $\lambda_1^y = -0.011$, $\lambda_2^y = 0.312$	3.5211E−2
14	$\lambda_1^x = 1.006$, $\lambda_3^x = 1.0032$, $\lambda_4^x = -1.3093$, $\lambda_1^y = -0.027$, $\lambda_2^y = 0.3028$	4.2477E−2

the obtained reconstruction results for the periodic and chaotic behaviors are reported in this section.

4.1 Periodic behavior reconstruction

A feasible approach to reconstruct periodic orbit from time series can be done by using Eq. (8). However, this approach is not computationally efficient and can be further enhanced by considering only the part of the time series once convergence is reached. In that case, it is enough to compute the distance between the closest periodic values of the attractors corresponding to the original and the reconstructed models. We applied this strategy to the reconstruction of the periodic orbits in Fig. 3. Figure 4 shows our results graphically: the superposition of the original (in blue) and the reconstructed (in red) periodic attractors. Note the good graphical matching between the original and the reconstructed periodic orbits, as the periodic points of both systems are visually very close to each other. This shows the ability of our method to reconstruct periodic orbits with good accuracy.

These good graphical results are also confirmed numerically. Table 2 shows the non-null parameter values of the reconstructed model for the periodic orbits in Fig. 4 along with their corresponding RMSE values. In addition, the table reports the cases of period-7 and period-14 orbits, which are also solved but not graphically displayed here because of limitations of space. As shown in the table, the RMSE is of order 10^{-2} for all periodic orbits in our benchmark, thus confirming numerically the good performance of the proposed method.

4.2 Chaotic behavior reconstruction

The reconstruction of chaotic orbits is more challenging, since the RMSE used for periodic orbits is no suitable for this case. Due to the strong sensitivity to initial conditions, any numerical error in the reconstruction process might amplify over the time, so the pairwise comparison of the time series is no longer possible here. The solution comes from the observation that, while the original and reconstructed time series can be shifted, so their temporal coherence is lost, the corresponding attractors should exhibit a

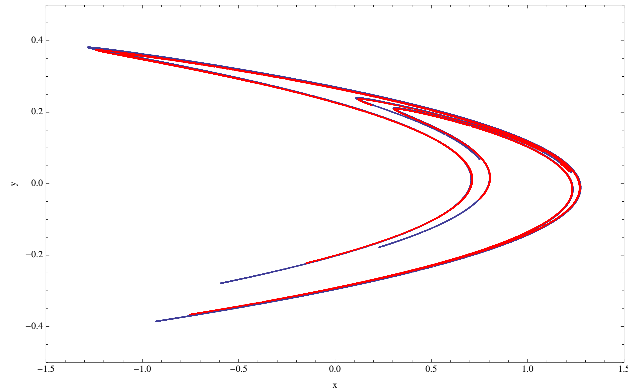

Figure 5: Superposition of the original (in blue) and reconstructed (in red) chaotic attractor of the Hénon map.

similar (but not necessarily identical) graphical structure. Therefore, in this case our approach consists of computing the set distance between both attractors, which is numerically computed as the ratio between their intersection and union sets.

We applied this strategy for the reconstruction of the chaotic attractor in Fig. 2 with satisfactory results. Figure 5 shows the superposition of the original (in blue) and reconstructed (in red) chaotic attractor of the Hénon map using our approach. As the reader can see, the original and reconstructed attractors exhibit a good matching, even although they are not fully identical. Numerically, the percentage of matching is 87.9% using the metric proposed in previous paragraph. We conclude that the method performs well also in this case and is able to reconstruct the general shape of the chaotic attractor with good accuracy.

5 CONCLUSIONS AND FUTURE WORK

In this paper, a new method is presented to reconstruct periodic and chaotic orbits of the 2D Hénon map from time series data. The method is based on a popular bio-inspired metaheuristic called cuckoo search algorithm with Lévy flights. The graphical and numerical results show that the method performs well and is able to recover the periodic and chaotic orbits of this map with good accuracy. We can conclude that this approach is very promising towards its applicability to this complex problem.

Regarding the ideas for future work in the field, we are interested to explore the extension of this method to more difficult scenarios, such as high-dimensional chaotic maps, and the case of continuous dynamical systems, described by sets of ordinary differential equations. A comparison of our results with those obtained with other techniques, ablation analysis without the Lévy flights, and the hybridization of our approach with a local search procedure are also part of our future work in the field.

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REFERENCES

[1] G.L. Baker, J.P. Gollub, *Chaotic Dynamics: An Introduction.* Cambridge University Press, Cambridge, UK, 1990.
[2] C.S. Bertuglia, F. Vaio, *Nonlinearity, Chaos, and Complexity: The Dynamics of Natural and Social Systems.* Oxford University Press, Oxford, UK, 2005.
[3] R.L. Devaney, *An Introduction to Chaotic Dynamical Systems, Third Edition.* CRC Press, Boca Raton, FL, 2022.
[4] V. Franceschini, Stable and unstable manifolds of the Hénon mapping. *Journal of Statistical Physics*, 25, 757–769, 1981.
[5] Z. Ghahramani, S. T. Roweis, Learning nonlinear dynamical systems using an em algorithm. In *Advances in neural information processing systems*, 431–437, 1999.
[6] C. Grebogi, E. Ott, J. A. Yorke, Chaotic attractors in crisis. *Physical Review Letters*, 48, 1507–1510, 1982.
[7] J. Güémez, J.M. Gutiérrez, A Iglesias, M.A. Matías, Stabilization of periodic and quasiperiodic motion in chaotic systems through changes in the system variables. *Physics Letters A*, 190 (5-6), 429–433, 1994.
[8] J. Güémez, J.M. Gutiérrez, A Iglesias, M.A. Matías, Suppression of chaos through changes in the system variables: transient chaos and crises. *Physica D: Nonlinear Phenomena*, 79 (2-4), 164–173, 1994.
[9] J.M. Gutiérrez, A. Iglesias, A Mathematica package for the analysis and control of chaos in nonlinear systems, *Computers in Science and Engineering*, 12 (6), 608–619, 1998.
[10] M. Hénon, A two-dimensional mapping with a strange attractor. *Communications in Mathematical Physics*, 50 (1), 69–77, 1976.
[11] M. Hénon, On the numerical computation of Poincare maps, *Physica D: Nonlinear Phenomena*, 5, 412–414, 1982.
[12] R. Hilborn, *Chaos and Nonlinear Dynamics: An Introduction for Scientists and Engineers.* Oxford University Press, Oxford, UK, 2001.
[13] G. E. Hinton, R. R. Salakhutdinov, Reducing the dimensionality of data with neural networks. *Science*, 313, 504–507, 2006.
[14] D. L. Hitzl, F. Zele, An exploration of the Hénon quadratic map. *Physica D: Nonlinear Phenomena.* 14 (3), 305–326, 1985.
[15] A. Iglesias, J.M. Gutiérrez, J. Güémez, M.A. Matías, Chaos suppression through changes in the system variables and numerical rounding errors, *Chaos, Solitons and Fractals*, **7**(8), 1305–1316, 1996.
[16] H. Jiang, H. He, State space reconstruction from noisy nonlinear time series: An autoencoder-based approach. In: *Proc. International Joint Conference on Neural Networks (IJCNN)*, IEEE, 3191–3198, 2017.
[17] M. Karl, M. Soelch, J. Bayer, P. Van der Smagt, Deep variational Bayes filters: Unsupervised learning of state space models from raw data. In: *Proc. International Conference on Learning Representations*, 1–13, 2017.
[18] Z. Lu, B.R. Hunt, E. Ott, Attractor reconstruction by machine learning. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, 28, Paper 061104, 2018.
[19] B. Lusch, J.N. Kutz, S.L. Brunton, Deep learning for universal linear embeddings of nonlinear dynamics. *Nature communications*, 9, 1–10, 2018.
[20] P. Mellodge, *A Practical Approach to Dynamical Systems for Engineers, 1st ed.*, Elsevier, Amsterdam, The Netherlands, 2015.
[21] K.R. Müller, A.J. Smola, G. Rätsch, B. Schölkopf, J. Kohlmorgen, V. Vapnik, Predicting time series with support vector machines. In: *Proc. of International Conference on Artificial Neural Networks*, 999–1004, Springer, 1997.
[22] S.S. Rangapuram, M. Seeger, J. Gasthaus, L. Stella, Y. Wang, T. Januschowski. Deep state space models for time series forecasting. In: *Proc. of the 32nd Int. Conf. on Neural Information Processing Systems*, ACM Press, 7785–7794 (2018).
[23] Q. She, Y. Gao, K. Xu, R.H. Chan, Reduced-rank linear dynamical systems. In:*Proc. Thirty-Second AAAI Conf. on Artificial Intelligence*, ACM Press, 4050–4057, 2018.
[24] C. Simó, On the Hénon-Pomeau attractor. *Journal of Statistical Physics.* 21 (4), 465–494, 1979.
[25] S.H. Strogatz, *Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry, and Engineering, 2nd ed.*, CRC Press, Boca Raton, FL, 2015.
[26] J.M.T. Thompson, H.B. Stewart, *Nonlinear Dynamics and Chaos, 2nd ed.*, John Wiley & Sons, Hoboken, NJ, 2002.
[27] Yang, X.S., Deb, S.: Cuckoo search via Lévy flights. In: *Proc. World Congress on Nature & Biologically Inspired Computing (NaBIC).* IEEE, 210–214, 2009.
[28] Yang, X.S., Deb, S.: Engineering optimization by cuckoo search.*Int. J. Mathematical Modelling and Numerical Optimization*, 1(4), 330–343, 2010.